

by the whole line thus produced and the part produced, with the square on half the line, is equal to the square on the line made up of the half and the part produced.

4. Prove that the angle at the centre of a circle is double the angle at the circumference on the same arc.

Show also that this proposition is true when the angle at the centre is greater than two right angles.

5. If a straight line touch a circle, and from the point of contact a straight line be drawn cutting the circle, the angles which this line makes with the tangent are equal to the angles in the alternate segments of the circle.

Two circles touch each other internally in A, and through the point of contact two chords AB and AC are drawn to the extremities of a diameter of either circle: AB and AC (produced if necessary) cut the other circle in P and Q: prove that PQ is a diameter of the second circle, and that it is parallel to BC.

6. If the sides of a triangle be bisected, and through the points of bisection perpendiculars be drawn to the sides, prove that these perpendiculars meet in a point.

7. Construct an isosceles triangle having each angle at the base double the third angle.

Show that your figure contains two triangles which satisfy the required condition, and that the third triangle has one angle three times as great as each of the others.

*Trigonometry.—Optional for Senior Civil Service. Time allowed: Three hours.*

1. Investigate a formula connecting the circular measure of an angle with its measure in degrees.

A Volunteer is shooting at a target 6ft. high, from a distance of 300 yards: calculate the number of seconds in the angle which the target subtends at his eye.

2. Express all the trigonometrical ratios of an angle in terms of the cosecant

If  $\operatorname{Cosec} \theta = \frac{a^2}{b^2}$ , and  $\tan \theta = \frac{b}{2a}$ , find the ratio of  $a$  to  $b$ .

3. Prove that  $\cos(x+y) = \cos x \cos y - \sin x \sin y$ .

Show that  $\cos(A+2B) \tan A + \sin(A+2B) - \sin A \cdot \sec 2(A+B) = \frac{\cos(3A+2B) \sin(A+2B)}{\cos A \cdot \cos 2(A+B)}$ .

4. What is meant by the *logarithm* of a number to a given base? and what by the *characteristic* of the logarithm?

Prove the rule for finding the characteristic by inspection.

Having given  $\log. 2 = \cdot 3010300$ ,  $\log. 3 = \cdot 4771213$ ,  $\log. 7 = \cdot 8450980$ , find the logarithms of

$\cdot 006$ ,  $14\cdot 7$ ,  $\left(\frac{3}{14}\right)^{\frac{1}{2}}$ ,  $(\cdot 015)^2$ .

5. Prove that, in any triangle,  $a = b \cos C + c \cos B$ .

Apply this to prove  $a(b^2 + c^2) \cos A + b(c^2 + a^2) \cos B + c(a^2 + b^2) \cos C = 3abc$ .

6. A surveyor finds that the sides of a quadrilateral field ABCD are, AB = 70 chains, BC = 60 chains, CD = 80 chains, DA = 90 chains, and that the diagonal AC is 110 chains. Calculate the area of the field, having given the logarithms in question 4, and also

$\log. 1\cdot 8973 = \cdot 2781360$

$\log. 3\cdot 5496 = \cdot 5501794$

$\log. 1\cdot 8974 = \cdot 2781589$

$\log. 3\cdot 5497 = \cdot 5501917$ .

7. Solve the equations—

$$\tan \theta + \cot \theta = \frac{4}{\sqrt{3}};$$

$$\tan 3\theta - \tan 2\theta + \tan \theta = 0.$$

*Maori.—Optional for Junior or Senior Civil Service.*

1. Translate into English the following:—

E mihi ana hoki au e pouri ana ki te ngaronga whakareretanga o nga manu kua kore nei, hei whakaahuareka i a tatou ki tona reo pai ina korero ratou i runga i nga rakau. Nga manu ataahua, whai tohu o te tau, kei hea ra? Me te riroriro, e waiata nei i tana waiata pai ina tae ki te aroaro mahanatanga o te tau—e ngarongaro katoa ana. E puta mai ana te aroha ki nga tangata kua riro; ko te ritenga ia o nga manu tohu o te tau e whakarongo tahi ai i a matou ki te tangi o aua manu te hunga kua mate atu ra. Kua riro ratou, ko te aroha kua waiho ki te hunga ora ngau kino ai i roto i te ngakau; he aroha mamae rawa e kore e taea e nga takuta te rongoa kia kore ai te aroha. He maha nga mate e ora i a ratou, tena ko te mamae aroha i roto i te ngakau e kore e ora i a ratou.

2. Translate into Maori the following:—

But the stars which marked the seasons of the year still abide and twinkle in the heavens, the stars which guided our fathers in the planting of their kumaras. There were four of these stars, Matariki (Pleiades), Tautoru (Orion), Puanga (Rigel), and Whakaahu. If the appearance of these betokened a favourable season, our fathers planted in September; if otherwise, they put off planting till October. I have alluded to the stars because we regard them with feelings of sadness, as having been the guides of our parents and our ancestors, who have passed away from this world. The birds of the forest which marked the seasons were the riroriro, the pipitori, the wharauora,