8. If a straight line be bisected, and produced to any point, the square of the whole line thus produced, and the square of the part of it produced, are together double of the square of half the line bisected, and of the square of the line made up of the half and the part produced.

ABC is a right-angled isosceles triangle. The hypotenuse BC is produced to any point D, and AD is joined. Show that $2AD^2 = BD^2 + CD^2$.

Euclid, Books I.-IV.—Optional for Senior Civil Service. Time allowed: 3 hours.

1. If one side of a triangle be produced, the exterior angle is greater than either of the two interior opposite angles.

2. If a straight line fall on two parallel straight lines it makes the alternate angles equal, and the exterior angle equal to the interior opposite angle on the same side of the line, and the two interior angles on the same side of the line together equal to two right angles.

Two circles are in the same vertical plane, and the highest point A of one is joined to the lowest point B of the other. Prove that the radii to the remaining intersections of AB with the circles are parallel.

3. If a straight line be divided into any two parts, the squares on the whole line and on one of the parts are together equal to twice the rectangle contained by the whole line and that part, together with the square on the other part.

4. If from any point within a rectangle straight lines are drawn to the angular points, the sum of the squares on one pair of lines drawn to opposite angles is equal to the sum of the squares on the other pair.

5. If a quadrilateral be inscribed in a circle its opposite angles are together equal to two right angles.

6. Angles in the same segment of a circle are equal to one another.

Find a point in a given straight line such that two given points subtend at it the greatest angle.

7. Inscribe a circle in a given triangle.

Show also how to describe a circle touching one side of a triangle and the other two produced.

Trigonometry.—Optional for Senior Civil Service. Time allowed: 3 hours.

1. Define the tangent of an angle, and trace the changes in magnitude and sign which the tangent undergoes as the angle increases from zero to two right angles.

2. Prove the relations,—

$\mathrm{Sec}^2\theta = 1 + \mathrm{Tan}^2\theta.$

$(\mathrm{Sec}^2 A + \mathrm{Tan}^2 A)(\mathrm{Cosec}^2 A + \mathrm{Cot}^2 A) = 1 + 2\,\mathrm{Sec}^2 A \,.\, \mathrm{Cosec}^2 A.$

$\mathrm{Vers}^2 A - 2\,\mathrm{Vers}\,A + \mathrm{Sin}^2 A = 0.$

3. Prove that—

$\mathrm{Tan}\,(180^\circ + A) = \mathrm{Tan}\,A.$

$\mathrm{Cos}\left(\frac{3\pi}{2} + A\right) = -\,\mathrm{Sin}\,A.$

4. Prove the relations—

$(\mathrm{Sin}\,\alpha + \mathrm{Sin}\,\beta)(\mathrm{Cos}\,\alpha + \mathrm{Cos}\,\beta) = \mathrm{Sin}\,(\alpha + \beta)\,[1 + \mathrm{Cos}\,(\alpha - \beta)].$

$\mathrm{Cot}\,3A \,.\, \mathrm{Tan}\,5A - \mathrm{Cot}\,3A \,.\, \mathrm{Tan}\,2A - \mathrm{Tan}\,5A \,.\, \mathrm{Tan}\,2A = 1.$

$\mathrm{Cos}\,A + \mathrm{Cos}\,3A + \mathrm{Cos}\,7A + \mathrm{Cos}\,9A = 4\,\mathrm{Cos}\,A \,.\, \mathrm{Cos}\,3A \,.\, \mathrm{Cos}\,5A.$

5. Show that in any triangle $\mathrm{Cos}\,A = \frac{b^2 + c^2 - a^2}{2bc}$, and deduce that $\mathrm{Tan}\,\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$ when s is half the sum of the sides.

6. Investigate the formulæ for completely solving a triangle when two sides and the included angle are given.

If $b = 7235$, $c = 1592$, $A = 50^\circ$, find B and C, having given

Log. 5·643 = ·7515101	Log. Tan 65° = 10·3313275
Log. 8·827 = ·9458131	Log. Tan 53° 53′ = 10·1368805
	Log. Tan 53° 54′ = 10·1371459.

7. At A and B the angles of elevation of an object, P, are observed to be α and β respectively. The distance between A and B is c, and the angle between AB and the line joining A to the foot of P is θ. Prove that the height, h, of P is given by the quadratic,

$$h^2\,(\mathrm{Cot}^2\beta - \mathrm{Cot}^2\alpha) + 2\,c\,h\,\mathrm{Cot}\,\alpha \,.\, \mathrm{Cos}\,\theta = c^2.$$

Mechanics.—Optional for Class D, and for Junior and Senior Civil Service. Time allowed: 3 hours.

1. Define *momentum, moment, mass, weight, centre of gravity, centre of pressure, specific gravity.*

2. Explain the principle of the composition of velocities.

A ship finds herself at noon 180 miles to the north, and 240 miles to the east, of the position which she occupied at noon of the previous day. Supposing her course to have remained unaltered, find her average rate of sailing per hour.

3. Define the units of *acceleration, force,* and *work.*

Find the velocity which a force equal to the weight of 5lb. will impart to a mass of 40lb. in 10 seconds.

Find, also, the work done by the force in imparting this velocity.