10. Simplify—

(1.) $\left(y - \frac{a^2 - xy}{x - y}\right)\left(x + \frac{a^2 - xy}{y - x}\right) + \left(\frac{a^2 - xy}{y - x}\right)^2$;

(2.) $\frac{a - x}{a + x} + \frac{4ax}{a^2 - x^2} + \frac{a + x}{a - x}$.

11. Show that—

(1.) $1 + n + \frac{1}{2}n(n + 1) + \frac{1}{6}n(n + 1)(n + 2) = \frac{1}{6}(n + 1)(n + 2)(n + 3)$;

(2.) $(a^2 + b^2 + c^2)(x^2 + y^2 + z^2) = (ax + by + cz)^2 + (bz - cy)^2 + (cx - az)^2 + (ay - bx)^2$.

Euclid.—For Class D, and for Junior Civil Service. Time allowed: 3 hours.

1. Explain the terms—postulate, proposition, problem, perpendicular, parallel, perimeter, polygon, projection.

2. If the three sides of one triangle be respectively equal to the three sides of another triangle, the two triangles shall be equal in every respect.

3. Prove that—

(1.) The straight lines which join the extremities of equal and parallel straight lines, towards the same parts, are themselves equal and parallel.

(2.) The straight lines which join the extremities of equal and parallel straight lines, towards opposite parts, bisect one another.

(3.) If two intersecting lines bisect one another, the lines joining their extremities form a parallelogram.

4. Triangles on the same base, and between the same parallels, are equal in area.

Triangles on the same base, but on opposite sides of it, are equal in area if the line joining their vertices be bisected by the common base.

5. On a given straight line to describe a parallelogram which shall be equal to a given triangle, and have one of its angles equal to a given angle.

6. Having given the sum of the diagonal and side of a square, to construct the square.

7. If a straight line be divided into two equal and also into two unequal segments, the sum of the squares on the two unequal segments is double the sum of the squares on half the line and on the line between the points of section.

If a straight line be divided into two segments, when is the sum of the squares on the two segments the least possible?

8. To describe a square which shall be equal to a given rectilineal figure.

Euclid, Books I.–IV.—For Senior Civil Service. Time allowed: 3 hours.

1. How many parts of one triangle must in general be given equal to the corresponding parts of another, in order that the equality of the remaining parts may be insured?

Prove that, if one side of one triangle be equal to one side of another, and the angles adjacent to the equal sides be also equal, each to each, then the remaining sides of the one are equal to the remaining sides of the other triangle.

2. Define a parallelogram. Prove that the opposite sides and angles of parallelograms are equal.

Prove that if the two diameters of a parallelogram are equal its angles must be right angles.

3. What is a rectangle?

Prove that, if a straight line be divided into two parts, the square on the whole line is equal to the sum of the rectangles contained by the whole line and each of the parts.

4. Show how to find the centre of a given circle.

Find the centre of a circle of which a portion only is given.

5. Define a segment of a circle; the angle in a segment.

Prove that the angle in a semicircle is a right angle, and the angle in a segment less than a semicircle is greater than a right angle.

6. Show how to draw a straight line through an external point to touch a circle.

Prove that two such straight lines can be drawn, and that they are equal in length.

7. Show how to inscribe a circle in a given triangle.

Prove that the three bisectors of the angles of a triangle meet in a point.

8. Construct an isosceles triangle having each of the angles at the base double of the vertical angle.

Mechanics.—For Class D, and for Junior and Senior Civil Service. Time allowed: 3 hours.

1. Define the terms *moment, momentum, couple, energy, power, fluid, metacentre.*

2. Define the units of force and work.

Find the force which must be applied to a mass of 1 ton in order to give it in 15 seconds a velocity of 60ft. per second. Find also the work done by the force in the last 5 seconds.

3. Prove that the velocity, v, of a body starting from rest under the action of a constant force is given by the formula $v^2 = 2fs$, where s is the distance which the body has traversed, and f is the constant acceleration.

A ball thrown vertically upwards returns to the hand after 5 seconds. Neglecting the resistance of the air, find the initial velocity of the ball, and the height to which it has risen.

4. Enunciate and prove the "polygon of forces."