8. Solve the equations—

$$(a)\ \frac{x-3}{x-1} + \frac{x-2}{x-5} = 2;$$

$$(b)\ \frac{a}{b+x} + \frac{a}{b-x} = b;$$

$$(c)\ \frac{x+1}{y} = \frac{a}{b} = \frac{y-1}{x}.$$

9. At a mayoral election one of the candidates polled fifteen votes less than half the constituency, beating his opponent, who polled one-third of the constituency, by 100 votes: find the whole number of electors, and the number that abstained from voting.

10. A debt, which might have been paid exactly with $2x$ sovereigns and $4y$ half-crowns, or with $8y$ sovereigns and x half-crowns, was paid with a £10 note, and y sovereigns and x half-crowns were received as change: what was the debt?

Algebra.—For Senior Civil Service. Time allowed: 3 hours.

1. If $a=3$, $b=2$, $c=1$, find the values of—
 (1.) $a^3-b^3-c^3-3abc$.
 (2.) $\sqrt{\tfrac{1}{2}(a^3+b^3+c^3-3abc)} - \sqrt[3]{a^3+b^3+c^3-a^2c}$.
2. Multiply x^3-2x^2+4x-8 by x^3-2x^2-4x+8, and divide the product by x^3+2x^2+4x+8.
3. Find the Highest Common Divisor of x^4+2x^2+9 and $x^4+5x^3+13x^2+17x+12$.
4. Multiply $x+y$ by x^2-xy+y^2, and resolve into its factors $(2a+3b)^3+(2a-3b)^3$.
5. Simplify—

$$(1.)\ \frac{\dfrac{x+y}{x-y}+\dfrac{x-y}{x+y}}{\dfrac{x+y}{x-y}-\dfrac{x-y}{x+y}} \times \frac{\dfrac{1}{x-y}-\dfrac{1}{x+y}}{\dfrac{1}{(x-y)^2}+\dfrac{1}{(x+y)^2}} \div \frac{\left(1+\dfrac{y}{x}\right)\left(1-\dfrac{y}{x}\right)}{x-\dfrac{y^2}{x}}.$$

$$(2.)\ \frac{x^4+5x^3+13x^2+17x+12}{x^4+2x^2+9}.$$

6. Prove that $(x+a)(x+b) = x^2+(a+b)x+ab$.
 Resolve into their factors—
 (1.) x^2-7x+6; (2.) x^2-6x-7; (3.) $6x^2-7x+1$; (4.) $(a+b+c)(bc+ca+ab) - abc$.
7. Find the continued product of x^2-xy+y^2, $x^4-x^2y^2+y^4$, and $x^8-x^4y^4+y^8$.
 Reduce to its simplest form—

$$\frac{x^3+y^3}{2(x^3-y^3)} + \frac{x^2-xy+y^2}{x^2+xy+y^2} + \frac{xy\left\{2(x^2+y^2)-xy\right\}}{x^4+x^2y^2+y^4} + \frac{x^3-y^3}{2(x^3+y^3)}.$$

8. Solve the equations—

$$(1.)\ \frac{2x+5}{6x+4} = \frac{x+4}{3x+7};$$

$$(2.)\ \begin{cases} \dfrac{3x+5}{4} - \dfrac{3x-y}{5} = \dfrac{y+1}{3} + \dfrac{4}{5}, \\[2ex] \dfrac{2x+5}{6y+4} = \dfrac{x+6}{3y+10}. \end{cases}$$

9. Divide the number 100 into four parts such that, if the second be increased by 10, the third by 8, and the fourth by 2, they each become equal to the first.

10. A passenger steamer, going up a certain river, travels from A to B in $8\tfrac{3}{4}$ hours. Coming down, it can go from C, a place further up the river than B, to A in the same time. There are eleven stopping-places between A and B, and, including B, nineteen between A and C. The distances between consecutive stopping-places are all equal, and the time occupied by each stoppage is a quarter of an hour. The stream flows at the rate of $4\tfrac{1}{2}$ miles per hour. Find the rate of the steamer in still water, and the distances between A, B, and C.

11. The number of candidates in a certain examination is a number of three digits, the first of which is equal to the sum of the other two. The number of failures is $\tfrac{2}{9}$ of the whole number diminished by unity. The last digit in the number of failures is zero, the middle digit is half the middle digit in the total number, and the first digit the same as the last in the total. If the digits in the total number be reversed, we obtain a number exceeding the number of failures by 25. Find the number of candidates.

12. Prove that—

$$4(a^2+ab+b^2)^3 - 27(a^2b+ab^2)^2 = (a-b)^2(2a^2+5ab+2b^2)^2.$$

Euclid.—For Class D, and for Junior Civil Service.

1. State Euclid's postulates.

From a given point draw a straight line equal to a given straight line.

How is the construction in this problem rendered necessary by the limitations of the postulates?

2. On the same base and on the same side of it there cannot be two triangles having their sides which are terminated at one extremity of the base equal to one another, and likewise those which are terminated at the other extremity equal to one another.

What use does Euclid make of this theorem?

3. The side opposite an angle of a triangle is greater than the side opposite a smaller angle.

Show that the perimeter of an isosceles triangle is greater than that of a rectangle of equal area.