

How would you verify this experimentally? Show that if a pavement has to be entirely made of tiles of the same shape and size, there are only three regular geometrical forms that will do.

4. The opposite sides and angles of a parallelogram are equal to one another, and the diagonal bisects the parallelogram.

P, Q, R, S are the middle points of the sides DA, AB, BC, CD of a quadrilateral ABCD : show that PQRS is a parallelogram, whose sides are parallel to the diagonals of ABCD, and whose area is half that of ABCD.

5. The complements of the parallelograms that are about the diagonal of any parallelogram are equal to one another.

A point P is taken on the diagonal AC of a parallelogram ABCD, and joined with B and D : show that the triangle PAB is equal in area to the triangle PAD, and that the triangles PCB and PCD are also equal.

6. If a straight line be divided into two equal parts, and also into two unequal parts, the rectangle contained by the unequal parts, together with the square on the line between the points of section, is equal to the square on half the line.

7. E is the middle point of AB, and C is any other point in AB : the squares on AC and CB are together double of the squares on AE and EC.

The sum of the squares on the sides of a parallelogram is equal to the sum of the squares on its diagonals.

8. AB and AC are two given straight lines : it is required to describe an equilateral triangle DEF, having its vertex D on AC, and its base EF of given length on AB.

Carefully discuss the logic of your method of solution.

Euclid, Books I.—IV.—For Civil Service Senior (Old Regulations). Time allowed : 3 hours.

1. The opposite sides and angles of a parallelogram are equal, and the diagonals bisect one another.

If two opposite sides of a parallelogram be bisected, and two lines be drawn from the points of bisection to opposite ends of one diagonal, these two lines trisect the other diagonal.

2. ABC is an obtuse angled triangle, having the obtuse angle at C ; squares are described on the three sides, and perpendiculars are drawn from A, B, C to the opposite sides, or the opposite sides produced ; these perpendiculars are produced to meet, in L, M, N, the opposite sides of the squares, or these sides produced, that have been described upon the sides to which the perpendiculars have been drawn : prove the parallelogram BL equal to the parallelogram BN.

What other parallelograms may be proved equal ?

3. Let ABCD be a square, and in AB, BC, CD, DA take four points, E, F, G, H, so that AE, BF, CG, and DH shall be equal to one another, and join E, F, G, H : prove that the figure EFGH is a square.

Hence, assuming II. 4, prove I. 47.

4. State and prove the proposition in the Second Book about the square on the side of a triangle subtending an acute angle.

If the base AB of a triangle be divided in D, so that $AD = 2 DB$, then $AC^2 + 2 BC^2 = AD^2 + 2 BD^2 + 3 CD^2$.

5. Describe a square equal to a given rectilineal figure.

6. If one circle touches another internally, the straight line joining their centres and produced passes through the point of contact.

A circle touches another circle, and also touches a radius of the latter circle at the middle point of that radius : prove that the radii of the circles are as 3 is to 8.

7. The opposite angles of a quadrilateral inscribed in a circle are together equal to two right angles.

State also and prove the converse of this proposition.

8. Two circles touch externally : prove that any line drawn through the point of contact cutting the circles cuts off similar segments.

9. Inscribe a regular pentagon in a given circle.

10. The area of a triangle is equal to half the rectangle contained by the sum of the sides and the radius of the inscribed circle.

11. If two opposite sides of a quadrilateral be produced to meet in P, and if G, H be the middle points of the diagonals of the quadrilateral, the triangle PGH is equal to a quarter of the quadrilateral.

Euclid and Trigonometry.—For Civil Service Senior (New Regulations). Time allowed : 3 hours.

1. Triangles on the same base and between the same parallels are equal.

Given a rectilineal figure of five sides, show how to describe a quadrilateral equal to it. Hence, show how a triangle may be constructed equal to any rectilineal figure.

2. State the enunciations of the propositions of the Second Book which refer to a straight line divided into any two parts. Give the corresponding algebraical formulae.

3. Three times the sum of the squares on the sides of a triangle is equal to four times the sum of the squares on the lines drawn from the angular points to the middle points of the sides.

4. The angles in the same segment of a circle are equal.

5. If a side of an isosceles triangle, having each of the angles at the base double of the angle at the vertex, be represented by unity, find the base.

5—E. 1A.