

6. The sides of a right-angled triangle are 3, 4, 5 inches in length: find the length of the perpendicular from the right angle.

7. Similar triangles are to one another in the duplicate ratio of their homologous sides.

There are two equilateral triangles, and a side of one is three times a side of the other: compare their areas.

8. Define the circular measure of an angle, a radian, and a degree.

Prove the formulae for $\sin(A+B)$ and $\cos(A-B)$, and find the value of $\tan 3A$ in terms of $\tan A$.

9. If A, B, C are the angles of a triangle, prove—

$$(i.) \cos 2A + \cos 2B + \cos 2C + 4 \cos A \cos B \cos C + 1 = 0.$$

$$(ii.) \sin \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$$

$$(iii.) \tan \frac{A-B}{2} = \frac{a-b}{a+b} \cot \frac{C}{2}$$

10. Define the logarithm of a number to a given base. What are the advantages of using 10 as a base?

$$\text{Prove } \log_a b = \frac{\log_x b}{\log_x a}$$

$$\text{Given } \log_3 3 = a, \log_5 5 = b, \log_7 7 = c: \text{ prove } \log_{10} 14 = \frac{1+abc}{1+ab}$$

11. Two sides of a triangle are in the ratio of 8 to 5, and the angle between them is 50° : find the remaining angles.

$$\text{Given } \log 3 = 0.4771213$$

$$\log 13 = 1.1139434$$

$$L \cot 25^\circ = 10.3313275$$

$$L \tan 26^\circ 19' = 9.6942478$$

$$L \tan 26^\circ 20' = 9.6945656$$

12. Twenty feet from the foot of a tower the angle of elevation of the top is three times the angle of elevation from a point 100 feet from the foot: find the height of the tower.

Trigonometry.—For Civil Service Senior (Old Regulations). Time allowed: 3 hours.

1. ABCD is a rectangle, AE is perpendicular to BD, and EF is perpendicular to BC: prove that $\tan BAF = \sin ABD \cos ABD$.

The angles of a triangle are as 2:3:4: express them in circular measure.

$$2. \text{ Prove } \cos^2 \theta + \cos^2 (60^\circ - \theta) - \sin \theta \sin (60^\circ - \theta) = \frac{5}{4}$$

3. Express $\frac{1}{4}(\sin \theta \sin 3\phi - \sin \phi \sin 3\theta)$ as the product of 4 sines.

4. Prove—

$$(i.) \frac{1 - \sin A \cos A}{\cos A (\sec A - \operatorname{cosec} A)} \times \frac{\sin^2 A - \cos^2 A}{\sin^2 A + \cos^2 A} = \sin A$$

$$(ii.) \tan^{-1} \frac{5}{7} + \tan^{-1} \frac{1}{8} = \frac{\pi}{4}$$

5. If A, B, C are the angles of a triangle, and a, b, c the sides opposite to them, prove—

$$(i.) \sin^2 A - \sin^2 B + \sin^2 C = 2 \sin A \cos B \sin C$$

$$(ii.) \tan A + \tan B + \tan C = \tan A \tan B \tan C$$

$$(iii.) (a^2 - b^2) \cot C + (b^2 - c^2) \cot A + (c^2 - a^2) \cot B = 0$$

6. Find an expression for all the angles which have a given tangent.

Find all the values of θ which satisfy the equation $(\sqrt{3} - 1) \tan \theta = 1 - \sqrt{3} \tan^2 \theta$

$$7. \text{ Given } \log 2 = 0.3010300$$

$$\log 3 = 0.4771213$$

$$\text{Find } x \text{ from the equation } 2^x + 2^{x-1} = 10$$

How many digits are there in 2^{30} ?

8. Find the values of the sine and cosine of an angle of a triangle in terms of the sides.

9. When two sides of a triangle and the angle opposite one of them are given, under what circumstances will there be two solutions, one solution, or no possible solution?

If $a = 42, b = 80, B = 60^\circ$, find A .

$$\text{Given } \log 2 = 0.3010300, \log 3 = 0.4771213, \log 7 = 0.845098$$

$$L \sin 27^\circ 2' = 9.6575423$$

$$L \sin 27^\circ 3' = 9.6577898$$

10. At a point in a horizontal plain the elevation of the top of a mountain above the horizontal plain is $25^\circ 12'$; at another point in the plain a mile further away in a straight line the elevation is $12^\circ 18'$: find the height of the mountain in feet.

$$L \sin 25^\circ 12' = 9.6292$$

$$L \sin 12^\circ 54' = 9.3488$$

$$L \sin 12^\circ 18' = 9.3284$$

$$\log 4.063 = 0.6088$$