3. Multiply together $a^2 - 2ab + 4b^2$, $a^2 + 2ab + 4b^2$, and $a^4 - 4a^2b^2 + 16b^4$.
4. Divide $6a^4 + 4b^4 - a^3b + 13ab^3 + 2a^2b^2$ by $2a^2 + 4b^2 - 3ab$.
5. Find the factors of—
   (1.) $x^4 - 7x^2 - 18$
   (2.) $1 + (b - a^2)x^2 - abx^3$
   (3.) $x^8 + x^4y^4 + y^8$
6. Explain and prove, as you would to a class of beginners, the following statements :—
   (1.) $a - (b - c) = a - b + c$
   (2.) $m(x + y + z) = mx + my + mz$

Use a diagram to illustrate this.

7. Simplify—
$$\frac{\dfrac{a}{n} - \dfrac{n-x}{a} + \dfrac{ax}{n^2 - nx}}{\dfrac{a}{n-x} + \dfrac{n-x}{a} + 2}$$
8. Prove that, if $b^2 = ac$ and $c^2 = bd$, then also $(b^3 + c^3)^2 = (a^3 + b^3)(c^3 + d^3)$
9. Simplify—
$$\frac{x^2 + x - 1}{x^3 - x^2 + x - 1} + \frac{x^2 - x - 1}{x^3 + x^2 + x + 1} + \frac{2x^3}{1 - x^4}$$
10. Solve the equations—
   (1.) $4[8x - 5(7 - 4x) + 9(6 - 3x) + 12x] = 7[20x - 2(7x - 10) - 2]$
   (2.) $\left.\begin{array}{l} ax + by = c \\ px + qy = r \end{array}\right\}$
   (3.) $\dfrac{x^2 + 9x + 18}{x + 6} - \dfrac{x^2 - 9x + 18}{x - 6} = 22 - \dfrac{x^2 + 3x - 18}{x - 3}$

11. A steamer takes 4 hours less time to travel down-stream from A to B than up-stream from B to A. The steamer would travel in still water at the rate of 15 miles an hour, and the stream flows at the rate of $4\frac{1}{2}$ miles an hour. Find the time taken on each journey, and the distance from A to B.

12. To the cube of x is added fifteen times x; from the sum is taken seven times the square of x; and the remainder is then divided by the excess of x over unity. Show that if x is greater than unity the result must be positive in sign.

Elementary Mathematics.—For Civil Service Junior. Time allowed: Three hours.

1. (i.) Simplify—
$$\left(-\frac{2x}{y}\right)^3 \times \left(\frac{7ab^2c}{8x^2y^2z}\right) \div \left(\frac{-8abc}{7xyz}\right)$$
and explain clearly the reasons for each step in the process.

(ii.) Divide $x^6 + x^5y - 12x^4y^2 + 19x^3y^3 + 15x^2y^4 - 14xy^5 + 2y^6$ by $x^2 + 4xy - 2y^2$

2. Find the H.C.F. of $x^4 + 5x^3 + 11x^2 + 13x + 6$ and $x^3 - x^2 - 3x - 9$ and the L.C.M. of $(x^2 - 1)^2$, $x^2 - 3x + 2$, and $x^2 - 6x + 9$

Explain the principles upon which your methods depend.

3. Factorize—
   (i.) $3x^3 - 8x^2y$ (ii.) $x^3 + x^2 + x + 1$ (iii.) $64 - 12x - x^2$
   (iv.) $\dfrac{x^2}{4} - \dfrac{x}{y} + \dfrac{1}{y^2}$ (v.) $a^2 + 2ab + b^2 - c^2$ (vi.) $x^3 - 8$
4. Solve these equations, and in each case check the answer :—
   (i.) $5(x + 3) = 7(9 - x)$
   (ii.) $\dfrac{2(x + 1)}{5} - 8 = \dfrac{3x}{16} - 1$
   (iii.) $\left\{\begin{array}{l} \dfrac{x + y}{2} = 4\frac{1}{2} \\ x - y = 1 \end{array}\right.$

5. A and B were candidates for election, and A was returned by a majority of 119. If $\frac{1}{10}$ of those who voted for A had refrained from voting, B would have been returned by a majority of 31. To secure the return of B, what percentage of those who voted for A would have been required to transfer their votes to B?

6. The angles at the base of an isosceles triangle are equal, and if the equal sides be produced the angles on the other side of the base shall also be equal to one another.

ABC is a triangle in which AB = AC, and O is the middle point of BC. Two points, E and F, are taken in AB and AC (produced if necessary) at equal distances from A: prove that OE and OF are equal and make equal angles with AB and AC.

7. Through a given point draw a straight line parallel to a given straight line.

Give as many different constructions as possible and prove each case. Which is the most practical method, and why?

8. The diagonals of a parallelogram bisect one another.

Through a given point, P, draw a line so that the parts of it intercepted between two given lines, AB and AC, may be bisected at P.

9. Prove the geometrical theorem corresponding to the algebraical formula $x^2 + y^2 = 2xy + (x - y)^2$.

Show that if a given straight line is divided into two parts the sum of the squares on the parts is least when the line is bisected.