

Euclid.—For Class D. Time allowed : Three hours.

1. If at a point in a straight line two other straight lines on opposite sides of it make the adjacent angles together equal to two right angles, these two straight lines shall be in one straight line.

Hence show that if OA, OB, OC, OD be four straight lines drawn in order from the point O, and if the angle AOB = the angle COD, and the angle AOD = the angle BOC, the straight lines AO, OC must be collinear, and in like manner BO, OD must be collinear.

2. If one side of a triangle be greater than another side, the angle opposite the greater side is greater than the angle opposite the less, and conversely.

Prove that of all the straight lines that can be drawn from an external point O to meet a straight line PQ, the perpendicular OM is the shortest.

3. If two triangles have two angles of the one equal respectively to two angles of the other, and the sides adjacent to the equal angles in each triangle equal to one another, then the triangles are equal in all respects. Prove this proposition by superposition, and also by the indirect method.

4. Explain how by cutting out paper you can show that any parallelogram is equal in area to the rectangle on the same base and of the same height.

Hence deduce the formulæ for (a) the area of a parallelogram, (b) the area of a triangle, (c) the area of a trapezium (*i.e.*, a quadrilateral with two sides parallel).

5. If a straight line be bisected and produced to any point, the rectangle contained by the whole line thus produced and the part produced shall be equal to the difference between the square on the line made up of the half and the part produced and the square on half the line bisected. Prove this geometrically and also algebraically. Name the geometrical figures in your diagram whose areas are expressed by the several terms of the algebraical formula obtained.

6. Describe a square equal in area to a given rectilineal figure.

7. D, E, F are the middle points of the sides BC, CA, AB of a triangle: show that the perimeter of the triangle ABC is greater than the sum of AD, BE, and CF, and less than twice the same sum.

8. ABCD is a quadrilateral: find a point P such that PA shall be equal to PC, and PB equal to PD.

9. ABCD is a parallelogram; the angle A is 60° , AB is 12 feet, AD is 10 feet: find to the nearest inch the length of AC. (You are not to use trigonometry.)

10. H is any point in the straight line AB; ABCD is a square on AB, and AFGH is a square on the opposite side of AH: show that DH is perpendicular to BF.

Geometry and Trigonometry.—For Civil Service Senior. Time allowed : Three hours.

1. Prove that the diagonals of a parallelogram bisect one another.

The perimeter of a triangle is greater than the sum of the medians and less than twice that sum.

2. Prove, by superposition or otherwise, that two triangles are equal (i) when they have two sides equal each to each and the angles between those sides equal; (ii) when they have two angles equal each to each and a side adjacent to the equal angles equal.

ABC is a triangle, and through A a straight line XY is drawn at right angles to the bisector of the angle BAC. Show that if P be any point whatever on XY, the perimeter of the triangle BPC is greater than that of ABC.

3. If a straight line is divided into two equal parts and also into two unequal parts, the rectangle contained by the unequal parts, and the square on the line between the points of section, are together equal to the square on half the line.

Describe a rectangle equal to the difference of two given squares.

4. Draw a tangent to a circle from a given point.

Through a given point, within or without a given circle, draw a chord of given length.

5. Prove, by superposition or otherwise, that in equal circles the angles that stand on equal arcs are equal.

If the two lines by which two vertically opposite angles are formed are chords of a circle, then either of the angles is equal to the angle subtended at the circumference by the sum of the two arcs that subtend the two angles.

6. If two triangles are equiangular they are similar.

P and Q are points in the circumference of a circle whose diameter is OA, and OL is drawn perpendicular to PQ. Show that the triangles OAQ, OLP are similar, and that $OA : OQ = OP : OL$.

7. Explain carefully what is meant by the circular measure of an angle, and find the relation between a radian and a degree.

Calculate the diameter of the sun, supposing that it subtends an angle of half a degree and that its distance is 90,000,000 miles.

8. Define the trigonometrical ratios of an angle, and prove that their values depend on the size of the angle and on nothing else. Find the trigonometrical ratios of 30° .

A man directly opposite a post on the other side of a stream walks 50 yards along the bank and finds himself directly opposite another post. If the line between the two posts subtends angles of 30° at both places of observation, find the width of the stream.

9. Prove :—

$$(i.) (1 + \cot A + \tan A) (\sin A - \cos A) = \frac{\sec A}{\operatorname{cosec}^2 A} - \frac{\operatorname{cosec} A}{\sec^2 A}$$