

$$(ii.) \sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$(iii.) \frac{\tan 5A + \tan 3A}{\tan 5A - \tan 3A} = 4 \cos 2A \cos 4A$$

10. Find a formula for all the angles that have a given cosine. Solve these equations:—

$$(i.) \sqrt{3} \sin x - \cos x = 1$$

$$(ii.) 2 \cos x \cos 3x + 1 = 0$$

11. Prove that in any triangle—

$$(i.) \sin A + \sin B + \sin C = 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$$

$$(ii.) a^2 = b^2 + c^2 - 2bc \cos A$$

$$(iii.) \cos \frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$$

12. Show how to solve a triangle when two angles and a side are given.

To determine the height of the top C of a mountain a base AB of 5,300 ft. was measured on the horizontal plane. The angle subtended at A by BC was observed to be $50^\circ 18'$, and that subtended at B by AC to be $112^\circ 32'$; also, the angle of elevation of C was observed at A to be $48^\circ 7'$. Find the height of the mountain, given: $L \sin 48^\circ 7' = 9.8719$; $L \sin 67^\circ 28' = 9.9655$; $L \sin 17^\circ 10' = 9.4700$; $\log 53 = 1.7243$; $\log 12.35 = 1.0917$.

Mechanics.—For Class D, and for Civil Service Junior. Time allowed: Three hours.

[Illustrate your answers with diagrams.]

1. State the proposition known as the parallelogram of forces.

Two forces equal to weights of 16 grams and 24 grams respectively act on a particle in directions that make an angle of 60° with each other. Find their resultant.

2. Describe, with the aid of diagrams, three systems of pulleys, and show what is the mechanical advantage in each case.

3. Show how to find the resultant of two parallel forces acting on a rigid body in the same direction.

Two men support the ends of a bar 1.2 metre long, to which a weight of 100 kilograms is attached at a point 50 centimetres from one end of the bar. Neglecting the weight of the bar, find the pressure experienced by each man.

4. Find the magnitude of the horizontal force that will just keep a weight of 50 lb. from sliding down a smooth inclined plane whose base is 4 ft. and height 3 ft.

5. Show how to find the centre of gravity of a triangular piece of cardboard of uniform thickness.

6. What is expressed by the symbol g ?

If a stone falls from a point 100 ft. above the ground, what is its velocity at the moment of reaching the ground?

7. Define the terms "momentum," "mass," "velocity," and "unit of force."

Establish the formula "momentum = mass $\times$ velocity."

Compare the unit of force that corresponds to 2 ft. as the unit of length with the unit of force that corresponds to 1 ft. as the unit of length.

8. Enunciate Newton's laws of motion, and give illustrations of each law.

9. If the specific gravity of ice is 0.925 and that of sea-water 1.025, how much of a piece of ice 3,000 cubic metres in volume will float above the surface of the sea?

10. Describe the Bramah press. On what principle is its construction based.

If the diameters of the two pistons be 3 centimetres and 40 centimetres, what weight can be supported on the larger piston by a pressure equal to a weight of 28 kilograms on the smaller?

11. When the mercurial barometer is standing at 765 mm., what would be the reading of an oil barometer, if the specific gravity of the oil is 0.925 and that of mercury 13.6?

12. When a barometer 36 in. high is standing at 30 in., $\frac{3}{4}$ cubic inch of air is introduced, and the mercury falls to 24 in. Find the sectional area of the tube.

Theoretical Mechanics.—For Civil Service Senior.

[Illustrate your answer with diagrams.]

1. What do you understand by uniformly accelerated motion?

A body moves with an initial velocity of V units of distance in one unit of time, which is increased by f units in every unit of time: establish the formula

$$v^2 = V^2 + 2fs$$

where v denotes the velocity at the end of t units of time, and s the distance traversed in the time t .

If a stone is thrown vertically upwards with an initial velocity of 29.25 metres a second, how high will it rise, and in what time will it return to its original level?

2. A stone is thrown with a velocity of 18 metres a second at an angle of 60° to the horizon from the top of a perpendicular cliff 30 metres high: at what distance from the base of the cliff will the stone reach the ground? Show how to find the position of the stone at the end of a given time.

3. Give a brief description of Attwood's machine.

Two equal weights of 6 oz. each are attached to the ends of a fine cord passing over a pulley. The weights are put in motion by adding to one of them a weight of 4 oz., which is removed without disturbing the motion after the weights have traversed a distance of 36 feet. Find the distance traversed in the next second.