and the length of PP_4 obtained by calculation from PP_1. Comparison of this value with the true value gives ϵ, while ϵ_0 is obtained from (3). Column (8) contains the cotangents of (5), (9) gives a_1 b_1.c_1, &c., and $\Sigma\ (a^2 + b^2 + c^2)$; from this column $h = c_1 + c_2 + c_3$ and $2k = \frac{1}{3}\ \Sigma\ (a^2 + b^2 + c^2)$ are obtained.

The equations for P and Q are now formed and solved. With these values of P and Q the corrections to the observed angles are calculated, and the values entered in column (6) and applied to the angles in (5), giving the final plane angles as shown in (7).

At this stage the work is checked by calculating PP_4 from PP_1, using the sines of the final plane angles in (7), and, as shown on the schedule, the calculated value of PP_4 agrees with the true value, thus proving the correctness of the work.

The triangles are solved using the sines in (13) and the results given in (15). The cotangents of (5) in (8) are taken from "Tafeln für Maschinenrechnen," by Dr. F. G. Gauss, published at Halle, A.S., by Von Eugen Strien, and checked by Chambers's tables. The natural sines of (5) in (11) are taken from tables of "Natural Sines and Cosines," by C. L. H. Max Jurisch, published at Cape Town, S.A., by Herrman Michaelis, and checked by Chambers's tables, the value of 1″ being entered in (10). The natural sines of (7) in (13) are again taken from tables by C. L. H. Max Jurisch, the first and second of each triangle being checked by adding algebraically (12), the third being checked by Chambers's tables. The work being thus checked at every point reduces the risk of numerical slips to a minimum.

In solving the triangles the usual practice is to start from the longest base-line and divide that length by the sine of the opposite angle, entering the value obtained underneath the three sines of each triangle in (13). This value is in turn multiplied by the sines opposite the required sides and the results entered in column (15), and so on for each triangle.

The notation used is as follows:—

Let l^1 = length of PP_4, calculated from PP_1, using the angles from column (5);
let l = true length of PP_4:
then $\epsilon = \frac{l - l^1}{l}$ radians,

ϵ_0 = sum of angles at P (from (3)) − 360°,
a_1 = cot angle 1, column (5),
β_1 = cot angle 2, column (5),
$a_1 = 2\ a_1 + \beta_1$,
$b_1 = -a_1 - 2\ \beta_1$,
$c_1 = -\ a_1 + \beta_1$,
$2k = \frac{1}{3}\ \Sigma\ (a^2 + b^2 + c^2)$,
$h = c_1 + c_2 + c_3$,
i = the number of triangles.

The equations for P and Q are—

$$h\ P + 2i\ Q + \epsilon_0 = 0$$
$$2k\ P + h\ Q + \epsilon = 0$$

The corrections to the angles are—

Angle 1 = $a_1\ P - Q$
Angle 2 = $b_1\ P - Q$
Angle 3 = $c_1\ P + 2Q$, &c.

The final plane angles are therefore equal to column (5) + column (6).

Observed Angles.

No.	Angle.	Observers.	No.	Angle.	Observers.
1	45° 58′ 28·96″	8 G.	6	81° 42′ 12·77″	6 G.
2	46 16 12·70	4 G.	7	62 42 11·58	10 G.
3	87 45 16·11	6 G.	8	45 59 13·71	9 G, 4 L.
4	55 29 28·71	7 G, 4 L.	9	71 18 40·00	6 G.
5	42 48 17·81	6 G, 2 L.			

	(17)	⅓ Δ Error.
45° 58′	28·96″	+ 1·413″
46 16	12·70	+ 1·413
87 45	16·11	+ 1·413
179 59	57·77	
Sph. ex.	2·009	
	− 4·239	

	(18)	⅓ Δ Error.
55° 29′	28·71″	+ 0·786″
42 48	17·81	+ 0·786
81 42	12·77	+ 0·787
179 59	59·29	
Sph. ex.	1·649	
	− 2·359	

	(19)	⅓ Δ Error.
62° 42′	11·58″	− 1·412″
45 59	13·71	− 1·412
71 18	40·00	− 1·413
180 00	05·29	
Sph. ex.	1·053	
	+ 4·237	

NOTES.—8 G refers to number of complete sets of observations by Mr. H. E. Girdlestone; 7 G, 4 L refers to combined number of complete sets of observations by Mr. H. E. Girdlestone and Mr. H. J. Lowe, the mean being obtained as follows:—

Mean of Mr. Girdlestone's sets = 55° 29′ 29·16″ × 7 = 204·12″
Mean of Mr. Lowe's sets = 55 29 27·91 × 4 = 111·64
11) 315·76
55.29.28.71